basis of a one-semester course. In recognition of this fact, that part of the book has been made available separately as a paperback volume. In view of the high cost of books these days, the author and the publisher are to be commended for this service to the public.

WOLFGANG WASOW

24 [2.35].—W. MURRAY, Editor, *Numerical Methods for Unconstrained Optimization,* Academic Press, New York, 1972, xi + 144 pp., 24 cm. Price $8.95.

During the last fifteen years the field of unconstrained optimization has experienced a phenomenal rate of growth. As in other fields that have grown at such a fast rate, it is rare to find a book that provides a coherent overview of the subject and also clearly describes the latest important research results. This is such a book, and it is a very welcome addition to numerical analysis, and in particular, optimization literature.

The book provides an excellent survey of unconstrained optimization methods, successfully presenting both theoretical results and practical matters such as computer implementation. The material covered is up-to-date and includes results obtained subsequent to the joint IMA/NPL conference in January 1971 at which the papers were originally presented. Considering the very active roles that all of the contributors to this book have played in extending the frontiers of optimization, this is not surprising. Moreover, this reviewer shares the view of the editor that "most" of the material presented will not become obsolete during the next few years.

As in any book containing the contributions of several authors, the style of the chapters varies considerably. On the whole, however, the book is extremely readable. A description of each chapter follows.

Chapter 1 – *Fundamentals,* authored by W. Murray, outlines some basic theory upon which subsequent chapters rely. This includes: definitions, necessary and sufficient conditions for a minimum, properties of quadratic and convex functions, and methods for minimizing functions of a single variable.

Chapter 2 – *Direct Search Methods,* authored by W. H. Swann, surveys methods which depend only upon values of the objective function; i.e., methods which do not use derivative information. Discussed here are the well-known "pattern search" method of Hooke and Jeeves, Rosenbrock's method, the Davies, Swann and Campey method, the simplex methods of Spendley, Hext, and Himsworth, and of Nelder and Mead, (not to be confused with the simplex method for linear programming), generalized Fibonacci search, and modifications of these methods. Random search, Box's technique of evolutionary operation and the technique of minimizing with respect to each independent variable in turn are also very briefly described. Just after publication of the book, Powell showed that the latter method can fail on differentiable functions.

Chapter 3 – *Problems Related to Unconstrained Optimization,* authored by M. J. D. Powell, is concerned with the solution of two types of problems via unconstrained optimization: nonlinear least-squares and constrained optimization problems. There is a clear and informative discussion of the Gauss-Newton and Marquardt methods and of modified versions of these for dealing with least-squares problems. There is also an excellent discussion of algorithms requiring only function values based upon the generalized secant method and the quasi-Newton approach. For constrained problems, transformation of variables, penalty function methods and Lagrangian methods are discussed. Practitioners who have such problems to solve should take special note of the section on Lagrangian methods, as recent work has shown this approach to be more promising than the standard penalty function or barrier type of approaches.

Chapter 4 – *Second Derivative Methods,* authored by W. Murray, is exclusively concerned with Newton's method and modifications of it. After briefly describing methods proposed by several other authors, Murray devotes the rest of the chapter to a numerically stable method of his own based upon Cholesky factorization. This appears to be an excellent method if the time required for computing the matrix of second derivatives is not excessive.

Chapter 5 – *Conjugate Direction Methods,* authored by R. Fletcher is, in this reviewer's opinion, the best introductory discussion of these methods in print. Methods described include those developed by Powell, Smith, Fletcher and Reeves, and Zoutendijk and the Partan method.

Chapter 6 – *Quasi-Newton Methods,* authored by C. G. Broyden surveys all of the well-known quasi-Newton, (variable metric), updating methods and families of updating formulas. Theoretical properties of these methods are discussed, with the principal emphasis on convergence results. For some methods, statements are made about computational experience.

Chapter 7 – *Failure, the Causes and Cures,* authored by W. Murray, attempts to provide some helpful hints to the practical optimizer. Besides some general remarks on rounding errors and numerical stability, there is a good discussion of these aspects with regard to quasi-Newton algorithms. Here, recent work on implementing these algorithms using the Cholesky factorization of an approximate Hessian, rather than the inverse of that matrix, is described. There are also some remarks on computer input and interpretation of output.

Chapter 8 – *A Survey of Algorithms for Unconstrained Optimization,* authored by R. Fletcher, documents several fully available Fortran IV and ALGOL 60 codes which implement some of the better optimization methods. The information given should be of considerable interest to problem solvers.

Finally, there is an appendix which sets forth several definitions and results in linear algebra without proof with which the reader needs to be familiar. The common mistake of referring to the Sherman and Morrison modification rule as Householder's rule is made here.

Some obvious typographical errors that were noticed are: first line below (3.6.4): the first "infinity" should be "minus infinity"; p. 69, equation for $g$ and (6.2.5): $f$ omitted; (4.12.1): $T_3$ inside both sets of parentheses should be $T_2$; second line above (7.3.2): (1.6.1) should be (1.5.1); p. 133, line 4: (8) should be (7); p. 136, third reference: pollution should be solution.

D. G.

25 [3, 7, 8, 10].– JÜRG NIEVERGELT, J. CRAIG FARRAR & EDWARD M. REINGOLD, *Computer Approaches to Mathematical Problems*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1974, xiii + 257 pp., 24 cm. Price $8.95 (clothbound).

This is a delightful book. It introduces the student to a medley of techniques, algorithms, and facts, which would be known by a well-educated computer scientist with a mathematical bent. Yet he (or she) probably acquired this material haphazardly over the years from courses, colloquia, and technical conversations.